

Probing dark energy with braneworld cosmology in the light of recent cosmological data

Miguel A. García-Aspeitia^{a,b} Juan Magaña^c Alberto Hernandez-Almada^d V. Motta^c

^aConsejo Nacional de Ciencia y Tecnología,
Av. Insurgentes Sur 1582. Colonia Crédito Constructor, Del. Benito Juárez C.P. 03940,
México D.F. México.

^bUnidad Académica de Física, Universidad Autónoma de Zacatecas, Calzada Solidaridad
esquina con Paseo a la Bufa S/N C.P. 98060, Zacatecas, México.

^cInstituto de Física y Astronomía, Facultad de Ciencias, Universidad de Valparaíso, Avda.
Gran Bretaña 1111, Valparaíso, Chile.

^dInstituto de Física, Universidad Nacional Autónoma de México, Circuito Exterior C.U.,
A.P. 20-364, México D.F. 04510, México.

E-mail: aspeitia@fisica.uaz.edu.mx, juan.magana@uv.cl, aalmada@fisica.unam.mx,
veronica.motta@uv.cl

Abstract. In this paper, we consider a simple brane model with a generic dark energy component which could drive the accelerated expansion at late times of the Universe. We use the Supernovae type Ia, $H(z)$, baryon acoustic oscillations, and cosmic microwave background radiation measurements to constrain the brane tension, which is the main observable of the theory. From the study, we found an important tension between the different data sets and evidence of no gravity modifications by the existence of an extra dimension. Although this specific braneworld model is not compatible with the current cosmological observations and offers no new insights into the dark energy problem, it is not ruled out either. Our results show the need to further test of the braneworld model with appropriate correction terms.

Keywords: Cosmology, braneworlds.

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1 Introduction

Several cosmological observations of Supernovae of the Type Ia at high redshift, show evidence of an accelerated expansion of the Universe at late times [1–3]. This intriguing feature also is supported by the observations of anisotropies in cosmic microwave background radiation (CMB) and baryon acoustic oscillations (BAO). In the standard cosmological scenario, the responsible for the Universe accelerated expansion is an entity which made up the $\sim 70\%$ of its total content and it is dubbed as dark energy [DE; 4]. Several models try to explain this late time cosmic trend [5–7], but the most favored candidate by the cosmological data is still the cosmological constant [CC; 4]. However, the latter shows conceptual and theoretical problems when assuming that the CC energy density comes from quantum vacuum fluctuations [8, 9]. Under this assumption, the theoretical prediction of the CC energy density differs ~ 120 orders of magnitude with the observational estimations [8, 9]. The other well-known difficulty of the CC is the coincidence problem which states why DE density is similar to that of dark matter (DM) today [8].

The CC problems have motivated to propose alternative candidates for DE being some of the most popular: the quintessence, phantom field, Chapylin gas, Holographic DE, etc., (see [10] for a DE models review). Most of these models can satisfactory mimic the CC dynamics. However, until now the origin of DE has been an open question and we need to explore other alternatives.

An interesting paradigm is to consider extra dimensions of space-time which could be the source of the current accelerated expansion. For instance, the Dvali-Gabadadze-Porrati (DGP) model [11] generates a natural accelerated expansion with a geometrical threshold associated to a five-dimensional space-time. In the same vein, another five-dimensional models like that proposed by Randall and Sundrum (RSI or RSII) [12, 13] drive a late cosmic acceleration adding a field with a characteristic equation of state (EoS), even allowing the structure formation [14] described by the traditional cosmological models. In addition, the RSI-II models provide a solution for the hierarchy problem and the problem associated with quantum vacuum fluctuations of the CC (see [15, 16] for more details).

In a covariant approach of the RSII models, the Einstein's field equations are modified assuming a five dimensional bulk with Schwarzschild-Anti'dSitter (S-AdS₅) geometry and a four dimensional manifold embedded in this bulk, called the *brane*. Note that for cosmological purposes the brane is considered as a FLRW structure, but in general it can take any geometry. Then, the main modifications to the Einstein field equations lie in three new tensors: the first one considers second order corrections to the energy-momentum tensor; the second tensor allows matter in the bulk and the latter takes into account non-local effects associated with the Weyl's tensor [17] (it is worth to note that non-local corrections are negligible in cosmological cases [15]). An important term in the theory is the brane tension, λ , which shows the threshold between the corrections that come from branes of those who belong to the traditional Einstein's equation. It is important to mention that these brane correction terms produce important extra dynamics that can be tested using the latest astrophysical or cosmological observations.

Our main goal is to investigate the effect of one extra dimension on the current dynamics of the Universe, mainly in the DE properties. Indeed, we focus our inquiry on what is the preferred equation of state (EoS) for DE in brane models? and what is the constraint for the brane tension from observations? In our model, DE is in the brane as well as the other components in our Universe (baryons, radiation, dark matter) with the only restriction that gravity is the only interaction that can overstep to the extra dimension. In addition, our condition for the dark energy EoS is that always must fulfill a generalized inequality shown in Ref. [18] to obtain an accelerated dynamic on the brane. As it happens in General Relativity (GR), DE is divided in quintessence ($-1 < \omega_{de} < -1/3$), CC ($\omega_{de} = -1$), and Phantom field ($\omega_{de} < -1$), [10] but parametrized by the extra dimensions.

It is also important to mention that recent results consider an accelerated expansion of the Universe in a RS frame, but assuming a geometrical point of view (see [19] as interesting example). However, as far as we know, there is not literature in a RS frame with a robust analysis of the dynamic of DE in a simple brane scenario; in particular, maintaining the EoS of dark energy as a free parameter and using recent observations as constraints. Other brane models are considerably more complex, being discarded by recent BAO data, generating an important tension with the other observations [20]. The idea behind of this study is to propose a simple model for modified Friedmann equations based in RS scenario that has all the basic components of the Universe, including a generic EoS for DE. If this simple model is discarded or exist tension with observations, it will be necessary to make important changes in the topology of the theory.

As we mentioned previously, the fundamental free parameter of this theory is the brane tension λ , which provides information about the extra terms which they have influence in the field equations. Even more, it is more useful assume the ratio between the density of the different components and the brane tension as: ρ/λ to establish the high ($\rho \gg \lambda$) and low ($\rho \ll \lambda$) energy limits; i.e. at late times of the cosmic evolution we expect that extra terms are subdominant in comparison with the canonical ones provided by GR. On the other hand, at early times the brane dynamics dominates over others in the Universe. Therefore, we are interested in constraining the brane tension and the dark energy EoS using the latest cosmological SNIa, $H(z)$, BAO and CMB data on brane equations.

From here, it is possible to organize the paper as follows: In Sec 2 we show the modified Einstein's equation by the presence of branes in RS scenario. We find the modified Friedman equation assuming matter, radiation and a generic DE as the Universe components; in addition, we get the deceleration parameter in terms of brane corrections. In Sec. 3 we realize

a robust analysis of the theory using various observations as $H(z)$ measurements, SNe Ia, BAO, and CMB. In Sec. 4, we present the results obtained with the analysis of the previous section and finally in Sec. 5 we give some conclusions and remarks. In what follows, we work in units in which $c = \hbar = 1$, unless explicitly written.

2 Brane cosmology

First of all, we introduce the Einstein's field equation projected onto the brane

$$G_{\mu\nu} + \xi_{\mu\nu} = \kappa_{(4)}^2 T_{\mu\nu} + \kappa_{(5)}^4 \Pi_{\mu\nu} + \kappa_{(5)}^2 F_{\mu\nu}, \quad (2.1)$$

where $T_{\mu\nu}$ is the four-dimensional energy-momentum tensor of the matter trapped in the brane, $G_{\mu\nu}$ is the classical Einstein's tensor and the rest of terms in the right and left sides of this equation are explicitly given by:

$$\kappa_{(4)}^2 = 8\pi G_N = \frac{\kappa_{(5)}^4}{6} \lambda, \quad (2.2a)$$

$$\Pi_{\mu\nu} = -\frac{1}{4} T_{\mu\alpha} T_{\nu}^{\alpha} + \frac{T T_{\mu\nu}}{12} + \frac{g_{\mu\nu}}{24} (3T_{\alpha\beta} T^{\alpha\beta} - T^2), \quad (2.2b)$$

$$F_{\mu\nu} = \frac{2T_{AB} g_{\mu}^A g_{\nu}^B}{3} + \frac{2g_{\mu\nu}}{3} \left(T_{AB} n^A n^B - \frac{{}^{(5)}T}{4} \right), \quad (2.2c)$$

$$\xi_{\mu\nu} = {}^{(5)}C_{AFB}^E n_E n^F g_{\mu}^A g_{\nu}^B. \quad (2.2d)$$

Here G_N is the Newton's gravitational constant, λ is the previously mentioned brane tension, $\kappa_{(4)}$ and $\kappa_{(5)}$ are the four and five-dimensional coupling constants of gravity respectively. The tensor $\Pi_{\mu\nu}$ represents the quadratic corrections on the brane generated by the energy-momentum tensor, $F_{\mu\nu}$ gives the contributions of the energy-momentum tensor in the bulk, which is projected onto the brane through the unit normal vector n_A . The tensor $\xi_{\mu\nu}$ provides the contribution of the five-dimensional Weyl's tensor projected onto the brane manifold [17].

To derive the Friedmann equations under the modified field equations, we consider a homogeneous and isotropic Universe in which a line element is given by

$$ds^2 = -dt^2 + a(t)^2 (dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2)), \quad (2.3)$$

where $a(t)$ denotes the scale factor. The recent Planck measurements [4] suggest a curvature energy density $\Omega_k \simeq 0$, thus we assume a flat geometry. We consider radiation and dark matter components as perfect fluids in the brane. We assume that the bulk has no matter component. By combining Eqs. (2.1) to (2.3), we obtain the modified Friedmann equation:

$$H^2 = \kappa^2 \rho_{eff}, \quad (2.4)$$

where

$$\rho_{eff} = \sum_i \rho_i \left(1 + \frac{\rho_i}{2\lambda} \right), \quad (2.5)$$

defining $\kappa^2 = 8\pi G/3 = \kappa_{(4)}^2/3$ the *renamed* gravitational coupling constant. $H = \dot{a}/a$ is the Hubble parameter, and ρ_i is the energy density for the radiation, dark matter and DE.

¹Notice that the latin letters take the values 0, 1, 2, 3, 4.

It is worth to notice that the low energy regime, i.e. the canonical Friedmann equation, is recovered when $\rho_i/2\lambda \rightarrow 0$. Crossed terms were not used in the Friedmann equation, i.e. there is not interaction between different species. In addition, if we consider, for instance, that the bulk black hole mass vanishes, the bulk geometry reduces to AdS_5 and $\rho_{e0} = 0$ [15, 21].

As a complement, we write the modified Friedmann equation in high energy regime as:

$$H_{high}^2 = \kappa^2 \sum_i \frac{\rho_i^2}{2\lambda}, \quad (2.6)$$

where it is assumed that $\rho_i/2\lambda \gg 1$. The latter is the high energy limit because we are assuming that the mean density of the fluids is much higher than the brane tension, so that any brane corrections in the equations of motion are highly suppressed by the brane energy scale. This limit is specially used in inflationary cosmology where the effects are more noticeable (for an excellent review of brane world inflation see [22]).

As mentioned before, we assume that the EoS of DE satisfies the constraint

$$\omega_{de} < -\frac{1}{3} \left[\frac{1 + 2\rho_{de}/\lambda}{1 + \rho_{de}/\lambda} \right], \quad (2.7)$$

to obtain an accelerated Universe and this value must be constrained by the cosmological data. Thus, the Friedmann equation can be written as:

$$H^2 = \kappa^2 \left[\frac{\rho_{0m}}{a^3} \left(1 + \frac{\rho_{0m}}{2\lambda a^3} \right) + \frac{\rho_{0r}}{a^4} \left(1 + \frac{\rho_{0r}}{2\lambda a^4} \right) + \frac{\rho_{0de}}{a^{3(1+\omega_{de})}} \left(1 + \frac{\rho_{0de}}{2\lambda a^{3(1+\omega_{de})}} \right) \right]. \quad (2.8)$$

Using the density parameters, $\Omega_i \equiv \rho_i/\rho_{crit}$, and redshift, Eq. (2.7) and (2.8) can be written as:

$$E(z)^2 \equiv \frac{H(z)^2}{H_0^2} = \Omega_{0m}(1+z)^3 + \Omega_{0r}(1+z)^4 + \Omega_{0de}(1+z)^{3(1+\omega_{de})} + \mathcal{M} \left[\Omega_{0m}^2(1+z)^6 + \Omega_{0r}^2(1+z)^8 + \Omega_{0de}^2(1+z)^{6(1+\omega_{de})} \right], \quad (2.9)$$

where $\Omega_r = 2.469 \times 10^{-5} h^{-2} (1 + 0.2271 N_{eff})$, $N_{eff} = 3.04$ is the standard number of relativistic species [23], $\Omega_{0de} \equiv 1 - \Omega_{0m} - \Omega_{0r}$, and

$$\mathcal{M} \equiv \frac{H_0^2}{2\kappa^2\lambda} = \frac{\rho_{crit}}{2\lambda}, \quad (2.10)$$

being H_0 the Hubble constant, and ρ_{crit} the Universe critical density. Notice that when $\mathcal{M} \rightarrow 0$, the canonical Friedmann equation with w_{de} is recovered. If $w_{de} = \omega_\Lambda = -1$, i.e. the DE is the CC, we obtain the traditional ΛCDM dynamics.

In addition, from Eq. (2.7) the DE EoS should satisfy the following constraint to obtain a late cosmic acceleration:

$$\omega_{de} < -\frac{1}{3} \left[\frac{1 + 4\mathcal{M}\Omega_{0de}(1+z)^{3(1+\omega_{de})}}{1 + 2\mathcal{M}\Omega_{0de}(1+z)^{3(1+\omega_{de})}} \right], \quad (2.11)$$

On the other hand, using Eq. (2.9) the deceleration parameter, $q(t) \equiv -\ddot{a}(t)/a(t)H(t)^2$, can be written in terms of redshift as

Parameter	Allowance
h	0.7324 ± 0.0174 (Gaussian)
$\Omega_b h^2$	0.02202 ± 0.00046 (Gaussian)
Ω_m	$[0, 1]$ (Uniform)
w_{de}	$[-2.5, 0]$ (Uniform)
\mathcal{M}	$[0, 0.5]$ (Uniform)

Table 1: Priors on the different parameters of the brane-world model. For h and $\Omega_b h^2$ we use the values given by [25] and [26] respectively.

$$q(z) = \frac{q_I(z) + \mathcal{M} q_{II}(z)}{E(z)^2}, \quad (2.12)$$

where

$$\begin{aligned} q_I(z) &= \frac{\Omega_{0m}}{2}(1+z)^3 + \Omega_{0r}(1+z)^4 + \frac{\Omega_{0de}}{2}(1+3\omega_{de})(1+z)^{3(1+\omega_{de})}, \\ q_{II}(z) &= 2\Omega_{0m}^2(1+z)^6 + 3\Omega_{0r}^2(1+z)^8 + \Omega_{0de}^2(2+3\omega_{de})(1+z)^{6(1+\omega_{de})}. \end{aligned} \quad (2.13)$$

In the same way, we recover the traditional behavior for $q(z)$ when $\mathcal{M} \rightarrow 0$, i.e. when brane effects are negligible.

3 Data and Methodology

To constrain the braneworld parameters we will perform a Markov Chain Monte Carlo (MCMC) analysis using the recent cosmological data of the Hubble measurements, SNIa, BAO, and CMB from Planck data release 2015. We assume a Gaussian likelihood $\mathcal{L} \propto \exp(-\chi^2/2)$, which depends on the model parameters. We consider Gaussian priors on h , and $\Omega_b h^2$ (see Table 1), and the only free parameters of the analysis are Ω_m , \mathcal{M} , and ω_{de} . The affine-invariant MCMC method implemented in emcee package [24] is used to find the confidence region of these free parameters. In the following we present the data and how the merit functions, χ , are constructed.

3.1 $H(z)$ measurements

The measurements of the expansion rate of the Universe as a function of redshift, i.e. $H(z)$, are widely used to test cosmological models. In this case, we use 34 data points compiled by [27] which span the redshift range $0.07 < z < 2.3$.

We start by writing the merit function, χ_H^2 , as:

$$\chi_H^2 = \sum_{i=1}^{34} \frac{[H_{th}(z_i) - H_{obs}(z_i)]^2}{\sigma_{H_i}^2}, \quad (3.1)$$

where $H_{obs}(z_i)$ is the observed Hubble parameter at z_i , σ_{H_i} is the error function, and $H_{th}(z_i)$ the theoretical value given by Eq. (2.9).

3.2 Type Ia Supernovae (SNe Ia)

We use the Lick Observatory Supernova Search (LOSS) sample containing 586 SNe in the range $0.01 < z < 1.4$ [28]. The relation between the distance modulus μ and the luminosity distance d_L is given by

$$\mu(z) = 5 \log_{10}[d_L(z)/\text{Mpc}] + \mu_0, \quad (3.2)$$

where μ_0 is a nuisance parameter and

$$d_L(z) = (1+z) \int_0^z \frac{dz'}{E(z')}. \quad (3.3)$$

The expression for $E(z)$ was presented in Eq. (2.9). After we marginalize over μ_0 , the SNIa constraints are obtained by minimizing the function $\chi_{\text{SNIa}}^2 = A - B^2/C$, where

$$\begin{aligned} A &= \sum_{i=1}^{586} \frac{[\mu(z_i) - \mu_{\text{obs}}]^2}{\sigma_{\mu_i}^2}, \\ B &= \sum_{i=1}^{586} \frac{\mu(z_i) - \mu_{\text{obs}}}{\sigma_{\mu_i}}, \\ C &= \sum_{i=1}^{586} \frac{1}{\sigma_{\mu_i}^2}. \end{aligned} \quad (3.4)$$

3.3 Baryon acoustic oscillations

Baryon acoustic oscillations are the signature of the interactions of baryons and photons in a hot plasma on the matter power spectrum in the pre-recombination epoch. Since BAO measurements are the standard rules, they are used as a geometrical probe to constrain cosmological parameters of modified gravity and dark energy models. The BAO data calibrated to the CMB at the end of the drag epoch provide estimations of the Hubble parameter and angular diameter distance, d_A , through the distance scale D_V defined as

$$D_V(z) = \frac{1}{H_0} \left[(1+z)^2 d_A(z)^2 \frac{cz}{E(z)} \right]^{1/3}, \quad (3.5)$$

where c is the speed light, and $d_A(z)$ is the Hubble-free angular diameter distance which relates to the Hubble-free luminosity distance through $d_A(z) = d_L(z)/(1+z)^2$.

The different surveys usually show BAO information in quantities related to D_V or the the comoving sound horizon, $r_s(z)$:

$$r_s(z) = c \int_z^\infty \frac{c_s(z')}{H(z')} dz', \quad (3.6)$$

where the sound speed $c_s(z) = 1/\sqrt{3(1 + \bar{R}_b/(1+z))}$, with $\bar{R}_b = 31500 \Omega_b h^2 (T_{\text{CMB}}/2.7\text{K})^{-4}$, and T_{CMB} is the CMB temperature. Here we use the BAO data shown in Table 2. $D_H(z) = c/H(z)$, and the redshift z_d at the baryon drag epoch is fitted with the formula proposed by [36],

$$z_d = \frac{1291(\Omega_m h^2)^{0.251}}{1 + 0.659(\Omega_m h^2)^{0.828}} \left[1 + b_1 (\Omega_b h^2)^{b_2} \right], \quad (3.7)$$

Quantity	z	BAO measurement	Survey
$d_z \equiv \frac{r_s(z_d)}{D_V(z)}$	0.106	0.336 ± 0.015	6dFGS [29]
d_z	0.44	0.0870 ± 0.0042	WiggleZ [30, 31]
d_z	0.6	0.0672 ± 0.0031	WiggleZ [30, 31]
d_z	0.73	0.0593 ± 0.0020	WiggleZ [30, 31]
d_z	0.15	0.2239 ± 0.0084	SDSS DR7 [32]
d_z	0.32	0.1181 ± 0.0022	SDSS-III BOSS DR11 [33]
d_z	0.57	0.0726 ± 0.0007	SDSS-III BOSS DR11 [33]
$\frac{D_H(z)}{r_s(z_d)}$	2.34	9.18 ± 0.28	SDSS-III BOSS DR11 [34]
$\frac{D_H(z)}{r_s(z_d)}$	2.36	9.00 ± 0.3	SDSS-III BOSS DR11 [35]

Table 2: BAO data from different surveys: six-degree-Field Galaxy Survey (6dFGS), WiggleZ experiment, Sloan Digital Sky Survey (SDSS) Data Release 7 (DR7), Baryon Oscillation Spectroscopic Survey (BOSS)-SDSS DR9, and BOSS-SDSS DR11.

where

$$b_1 = 0.313 (\Omega_m h^2)^{-0.419} \left[1 + 0.607 (\Omega_m h^2)^{0.674} \right], \quad (3.8)$$

$$b_2 = 0.238 (\Omega_m h^2)^{0.223}. \quad (3.9)$$

The function-of-merit for all the BAO data points, χ_{BAO}^2 is

$$\chi_{BAO}^2 = \chi_{6dFGS}^2 + \chi_{WiggleZ}^2 + \chi_{DR7}^2 + \chi_{DR11A}^2 + \chi_{DR11B}^2, \quad (3.10)$$

where

$$\begin{aligned} \chi_{6dFGS}^2 &= \left(\frac{d_z(0.106) - 0.336}{0.015} \right)^2, \\ \chi_{WiggleZ}^2 &= \left(\frac{d_z(0.44) - 0.0870}{0.0042} \right)^2 + \left(\frac{d_z(0.6) - 0.0672}{0.0031} \right)^2 + \left(\frac{d_z(0.73) - 0.0593}{0.0020} \right)^2, \\ \chi_{DR7}^2 &= \left(\frac{d_z(0.15) - 0.2239}{0.0084} \right)^2, \\ \chi_{DR11A}^2 &= \left(\frac{d_z(0.32) - 0.1181}{0.0023} \right)^2 + \left(\frac{d_z(0.57) - 0.0726}{0.0007} \right)^2, \\ \chi_{DR11B}^2 &= \left(\frac{\frac{D_H(2.34)}{r_s(z_d)} - 9.18}{0.28} \right)^2 + \left(\frac{\frac{D_H(2.36)}{r_s(z_d)} - 9.00}{0.3} \right)^2. \end{aligned} \quad (3.11)$$

$$(3.12)$$

3.4 CMB: Planck 2015 measurements

The anisotropy measurements in the temperature of CMB radiation provide narrow constraints on cosmological parameters. A useful method to obtain cosmological constraints without performing a complete perturbative analysis is to reduce the full likelihood information to a few parameters, mainly: the acoustic scale, l_A , the shift parameter, R , and the

decoupling redshift, z_* [37]. Although these distance posteriors could lead to biased constraints when are used in modified gravity models [20], as a first approach, we constrain the braneworld model parameters using the R , l_A , and z_* values for a flat w CDM obtained from Planck measurements ($R = 1.7492 \pm 0.0049$, $l_A = 301.787 \pm 0.089$, $z_* = 1089.99 \pm 0.29$ [38]). Planck also provide the following inverse covariance matrix, Cov_{Pl}^{-1} , of these quantities

$$\text{Cov}_{Pl}^{-1} = \begin{pmatrix} 162.48 & -1529.4 & 2.0688 \\ -1529.4 & 207232 & -2866.8 \\ 2.0688 & -2866.8 & 53.572 \end{pmatrix}. \quad (3.13)$$

The merit function for the Planck data is constructed as

$$\chi_{Pl}^2 = X^T \text{Cov}_{Pl}^{-1} X, \quad (3.14)$$

where

$$X = \begin{pmatrix} l_A^{th} - l_A \\ R^{th} - R \\ z_*^{th} - z_* \end{pmatrix}, \quad (3.15)$$

the superscripts th refer to the theoretical estimations. The shift parameter is defined as [39]

$$R = \frac{\sqrt{\Omega_m H_0^2}}{c} r(z_*), \quad (3.16)$$

where z_* can be estimated using [40],

$$z_* = 1048[1 + 0.00124(\Omega_b h^2)^{-0.738}][1 + g_1(\Omega_m h^2)^{g_2}], \quad (3.17)$$

and

$$g_1 = \frac{0.0783(\Omega_b h^2)^{-0.238}}{1 + 39.5(\Omega_b h^2)^{0.763}}, \quad g_2 = \frac{0.560}{1 + 21.1(\Omega_b h^2)^{1.81}}. \quad (3.18)$$

The acoustic scale is defined as

$$l_A = \frac{\pi r(z_*)}{r_s(z_*)}, \quad (3.19)$$

where r is the comoving distance from the observer to redshift z given by

$$r(z) = \frac{c}{H_0} \int_0^z \frac{dz'}{E(z')}. \quad (3.20)$$

4 Results

In all our analysis, a total of 3500 steps with 500 walkers are generated and it takes 500 (burn-in) steps to stabilize our estimations. Table 3 summarizes the constraints and Figure 1 shows the 1D marginalized posterior distributions and 2D contours at 68%, 95%, and 99% confidence levels for each data set. When the Hubble measurements are used, a lower chi-square is obtained indicating an overfitting of the data. Although the value of w_{de} is consistent with the CC, Ω_m is lower than the standard prediction. Additionally, the low value of \mathcal{M} suggests that no corrections to the cosmological dynamics of the Universe is needed. We note that the SNIa data provide a good fit of the model parameters, the dark energy EoS is consistent with the CC at 2σ level and the best fit for \mathcal{M} favors the gravity modification by

Data set	χ^2	Ω_m	w_{de}	$\log(\mathcal{M})$	h	$\lambda(h^2 \times 10^{-10} \text{eV}^4)$
$H(z)$	18.19	$0.21^{+0.02}_{-0.03}$	$-1.00^{+0.11}_{-0.12}$	$-1.82^{+0.48}_{-0.63}$	$0.72^{+0.01}_{-0.01}$	$2.50^{+2.19}_{-0.96}$
SN Ia	574.73	$0.13^{+0.06}_{-0.07}$	$-0.81^{+0.07}_{-0.10}$	$-0.69^{+0.28}_{-0.56}$	$0.72^{+0.01}_{-0.01}$	$0.80^{+0.60}_{-0.19}$
BAO	5.46	$0.20^{+0.04}_{-0.07}$	$-0.53^{+0.13}_{-0.19}$	$-10.96^{+0.88}_{-0.84}$	$0.73^{+0.01}_{-0.01}$	$23306.3^{+51739}_{-13639.5}$
CMB	10.87	$0.29^{+0.01}_{-0.01}$	$-1.12^{+0.06}_{-0.06}$	$-16.93^{+0.75}_{-0.85}$	$0.73^{+0.01}_{-0.01}$	$9124570^{+12223730}_{-4814430}$

Table 3: Best values and its errors for Ω_m , w_{de} , and $\log(\mathcal{M})$ estimated from $H(z)$, SN Ia, BAO and CMB data. The brane tension values are calculated (see Eq. (2.10)) assuming a critical density $\rho_{crit} = 8.10h^2 \times 10^{-11} \text{eV}^4$.

an extra dimension. Nevertheless, the SN Ia data give an extremely low value for the matter content, which is ruled out by other observations. The BAO probe give a lower Ω_m bound, a w_{de} value consistent with quintessence, and a negligible \mathcal{M} value which indicates that the cosmological dynamics of the Universe is not affected by an extra dimension. The CMB data provide strong constraints on Ω_m and w_{de} , which are consistent the Λ CDM paradigm, and no gravity modification is preferred. As mentioned before, the compressed CMB data used here may not be appropriate to our braneworld model and these bounds would be biased.

Our results show an important tension in the \mathcal{M} constraints, which is directly related to the modifications to the Friedmann equation and hence to the Universe dynamics by an extra dimension. While the low-redshift data ($H(z)$ and SN Ia) suggest slightly evidence of gravity modifications, the high-redshift data (BAO and CMB) rule out an extra dimension in the Universe. On the other hand, although all data sets provide w_{de} bounds consistent with the CC, it is important to investigate whether it satisfies the late-time cosmic acceleration constraint given by the Eq. (2.11). For each cosmological data, we plot the confidence contours for the parameters with and without this constraint, finding differences only for the BAO limits. Figure 2 shows these regions. Notice that our best fits satisfy the Eq. (2.11) implying a late-time cosmic acceleration. Furthermore, to confirm this accelerated expansion, we reconstruct the deceleration parameter (Eq. (2.12)). Figure 3 shows the cosmological evolution of $q(z)$ vs. redshift using different cosmological constraints in two cases: using the \mathcal{M} best fit (solid lines) and $\mathcal{M} = 0$ (dashed lines). Notice that the Universe passes from a decelerated to an accelerated phase at $z = 0.9, 1.05, 0.69$, and 0.68 for the $H(z)$, SN Ia, BAO, and CMB constraints respectively. However, for the BAO constraints, $q(z)$ reaches a value ~ -0.13 at $z = 0$ which is in tension with the current estimations. The cosmic evolution of $q(z)$ also confirms that the cosmological data (except the SN Ia observations) prefer no modifications to the Friedmann equations by an extra dimension, i.e. the Universe has an accelerated expansion at late times without needing extra dimensions.

Finally, we emphasize the great discrepancy between the cosmological results found using brane tension and previous results obtained from nucleosynthesis [18] and stellar dynamics [41–44]. Brane tension is constrained as $\lambda > 1 \text{ MeV}^4$ in the first one and $\lambda \gtrsim 80 \text{ MeV}^4$ in the second one, showing the enormous difference between the results. In general, this comparison suggests that braneworld effects are noticeable mainly at high energy events like in the last stages of stellar dynamics (neutron star and black hole formation) or in the first stages of Universe evolution (inflation and super-unification epochs).

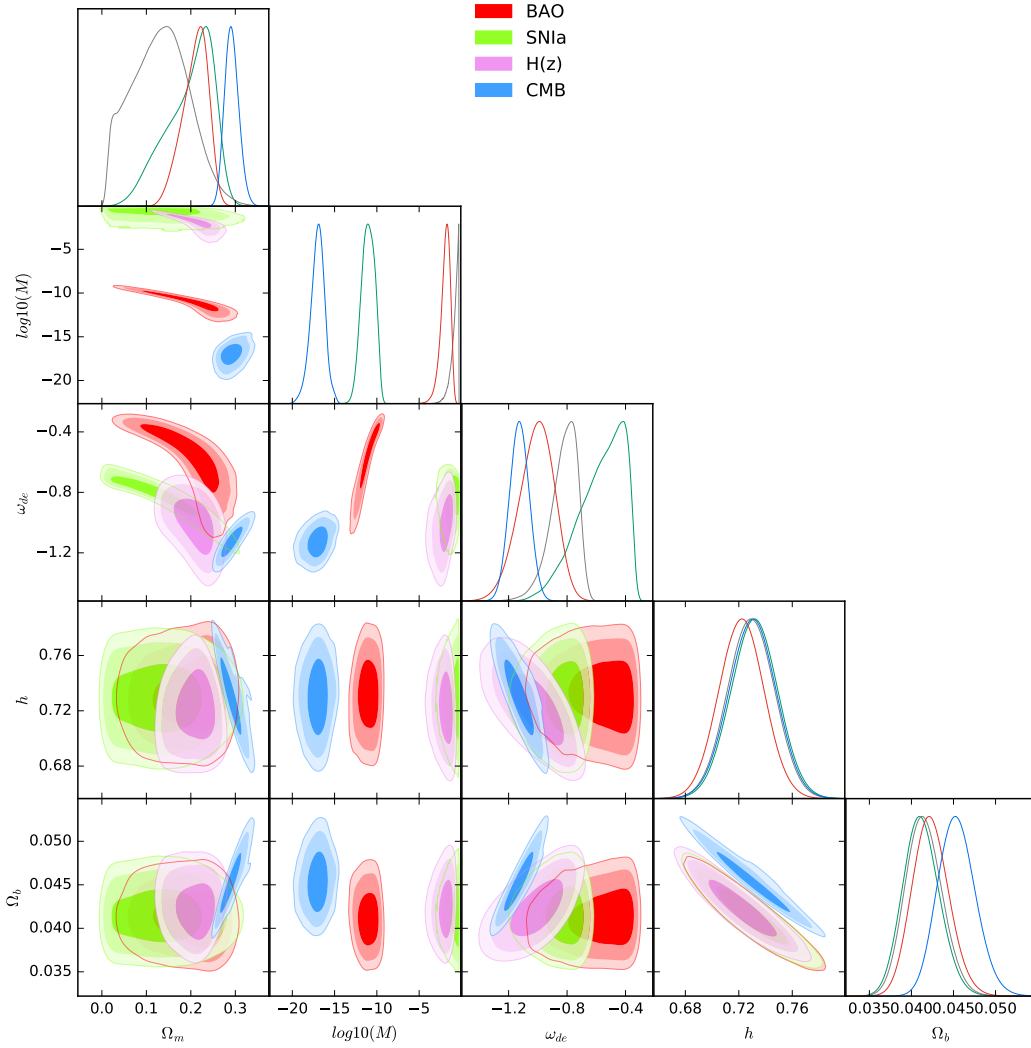


Figure 1: 1D marginalized posterior distributions and 2D contours at 68%, 95%, and 99% confidence levels for the Ω_m , $\log(\mathcal{M})$, w_{de} , h , Ω_b parameters using $H(z)$, SNIa, BAO, and CMB data.

5 Conclusions and Remarks

Brane theory is an interesting paradigm to solve many fundamental problems in Particle Physics and Cosmology, being an important candidate to extend GR. In this paper, we explored the consequences in the cosmic acceleration by considering a generic dark energy in a Randall-Sundrum braneworld scenario. We derived the modified Friedmann equations governing the dynamics of the Universe to investigate whether the current cosmological observations suggest such gravity corrections. We put constraints on the dark matter density parameter, dark energy EoS, and the brane tension using the latest observational data ($H(z)$, SNIa, BAO, and CMB from Planck data release 2015).

We found an important tension in the braneworld parameters, mainly in the brane tension (whose value is encoded in the \mathcal{M} parameter), using the different cosmological data. While the high-redshift data (BAO and CMB) preferred no gravity modifications, the low-

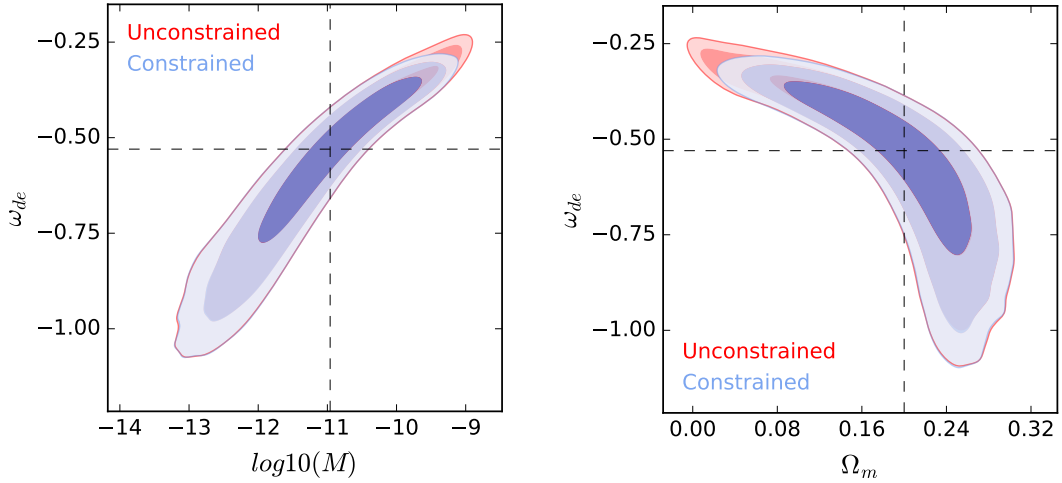


Figure 2: Comparison of contours (un)constrained to Eq. (2.11) for BAO data: w_{de} vs $\log_{10}(M)$ (left) and w_{de} vs Ω_m (right). In both figures, the non-constrained best fit is represented with dashed lines.

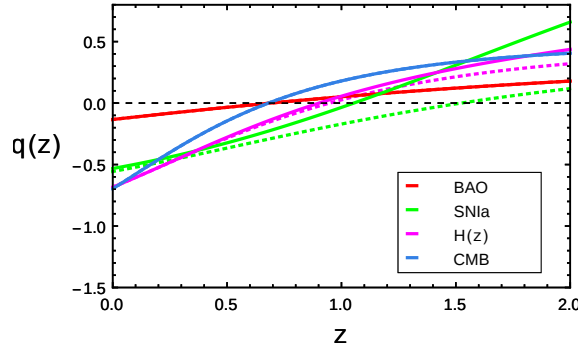


Figure 3: Reconstruction of the deceleration parameter $q(z)$ using the constraints from $H(z)$, SNIa, BAO and CMB data. We plot two cases: using the \mathcal{M} best fit (solid lines) and $\mathcal{M} = 0$ (dotted lines).

redshift data ($H(z)$ and SNIa) slightly suggest there is an extra dimension. It is worth to note that the constraints derived of the $H(z)$ probe would be ambiguous because it overfitting the data and the CMB constraints would be biased because the method used may not be appropriate for braneworld models. Actually, only the SNIa data favors gravity modifications by an extra dimension, however the dark matter content estimated is ruled out by other measurements. We reconstructed the deceleration parameter using the best fit for each data set and found that the dark energy component drives to a late-time cosmic acceleration independently of gravity modifications by an extra dimension. By comparing our constraints for the brane tension with previous studies using nucleosynthesis bounds, we found that there are important differences in both limits [18]. However, the astrophysical probes give brane tension constraints in good agreement with the nucleosynthesis results [41–44].

Finally, our results offer a low confidence in this specific braneworld model. Under this

scenario, we suggest the need for an appropriate extension of the modified FLRW equations for braneworld models, for instance, by considering the crossed terms which take into account coupling between the different Universe components. This future work will be presented elsewhere.

Acknowledgments

MAG-A acknowledges support from SNI-México and CONACyT research fellow. J.M. acknowledges support from FONDECYT 3160674. A.H. acknowledges CONACyT postdoctoral fellow; Instituto Avanzado de Cosmología (IAC) collaborations. V.M. acknowledges support from Centro de Astrofísica de Valparaíso.

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